Midterm Review

CS 103

Winter 2020

Outline

- Proof Techniques
- Set theory
- First Order Logic
- Relations
- Graph theory and functions

Proof Techniques

Proof Techniques

- Proving Implications

Proof Techniques

- Proving Implications
- Direct Proof

Proof Techniques

- Proving Implications
- Direct Proof
- Contrapositive

Proof Techniques

- Proving Implications
- Direct Proof
- Contrapositive
- Contradiction

Proof Techniques

- Proving Implications
- Direct Proof
- Contrapositive
- Contradiction
- Universal versus Existential Statements

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive?

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$."

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \emptyset$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \emptyset$."

Here's one way to start off the proof:

Consider arbitrary sets S and T.

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \emptyset$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \emptyset$."

Here's one way to start off the proof:

Consider arbitrary sets S and T.

We will prove by contrapositive that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \emptyset$.

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$."

Here's one way to start off the proof:

Consider arbitrary sets S and T.

We will prove by contrapositive that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$.

🤔 Is this a good/correct way to start?

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \emptyset$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \emptyset$."

Here's one way to start off the proof:

Consider arbitrary sets S and T.

We will prove by contrapositive that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \emptyset$.

🤔 Is this a good/correct way to start? NO! 😢

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$."

Here's a good way to start off the proof:

We will prove the contrapositive of this statement and show that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$.

Consider arbitrary sets S and T …

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$."

Here's a good way to start off the proof:

We will prove the contrapositive of this statement and show that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$.

Consider arbitrary sets S and T ___________________.

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$."

Here's a good way to start off the proof:

We will prove the contrapositive of this statement and show that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$.

Consider arbitrary sets S and T such that $S \neq T$. ←---- "ASSUME"

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$."

Here's a good way to start off the proof:

We will prove the contrapositive of this statement and show that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$.

Consider arbitrary sets S and T such that $S \neq T$. ←---- "ASSUME"

←---- "W.T.S."

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

Before we begin…

What's the contrapositive? "For any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$."

Here's a good way to start off the proof:

We will prove the contrapositive of this statement and show that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$.

Consider arbitrary sets S and T such that $S \neq T$. ←---- "ASSUME"

We will show that $S \Delta T \neq \varnothing$. ←---- "W.T.S."

Set Theory

(Fall 2019) Prove the following theorem by contrapositive:

For any sets S and T, if $S \Delta T = \varnothing$, then $S = T$.

We will prove the contrapositive of this statement and show that for any sets S and T, if $S \neq T$, then $S \Delta T \neq \varnothing$.

Consider arbitrary sets S and T such that $S \neq T$. ←---- "ASSUME"

We will show that $S \Delta T \neq \varnothing$. ←---- "W.T.S."

…

First Order Logic

First Order Logic

- Predicates

First Order Logic

- Predicates

- Logical Connectives
    - AND, OR, IMPLIES, BICONDITIONAL, NEGATION

First Order Logic

- Predicates

- Logical Connectives
  - AND, OR, IMPLIES, BICONDITIONAL, NEGATION

- Quantifiers
  - Universal and Existential

First Order Logic

- Predicates
- Logical Connectives
  - AND, OR, IMPLIES, BICONDITIONAL, NEGATION
- Quantifiers
  - Universal and Existential
- Negation

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

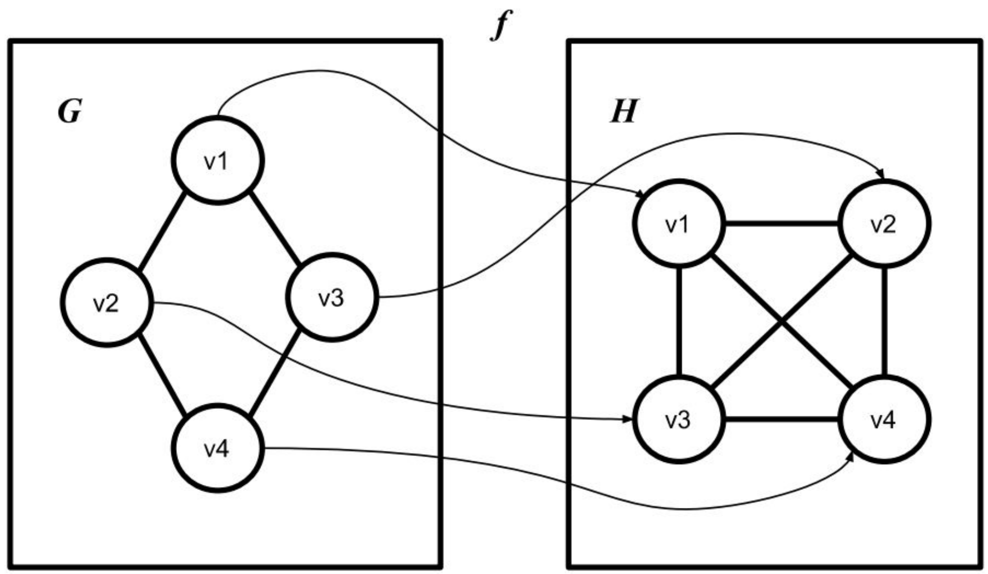
$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

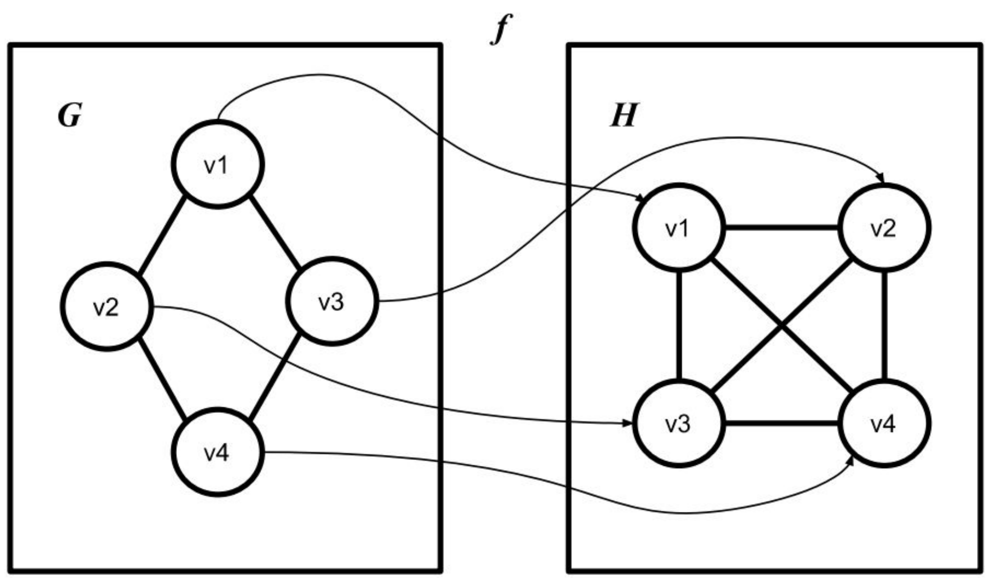


Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

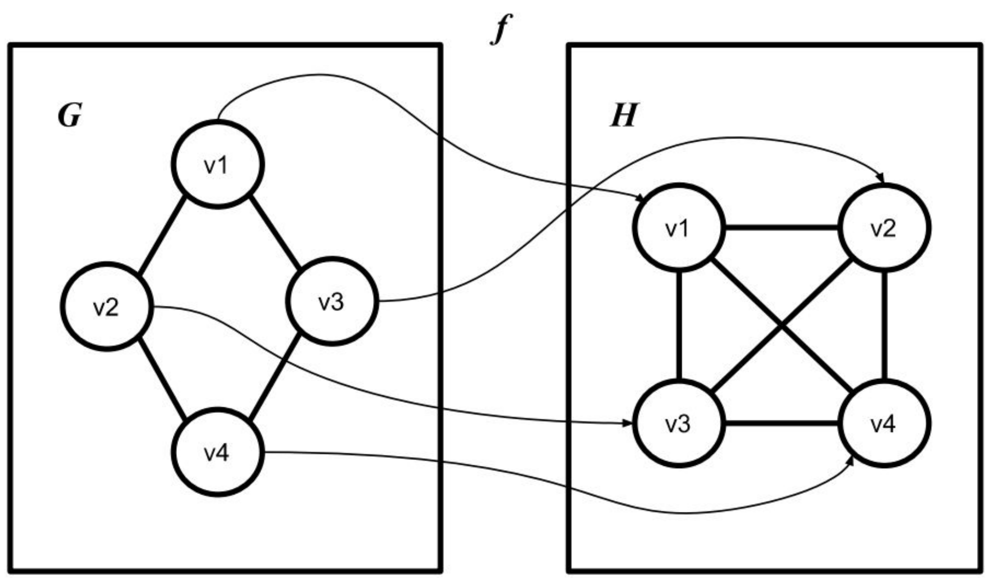


Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$



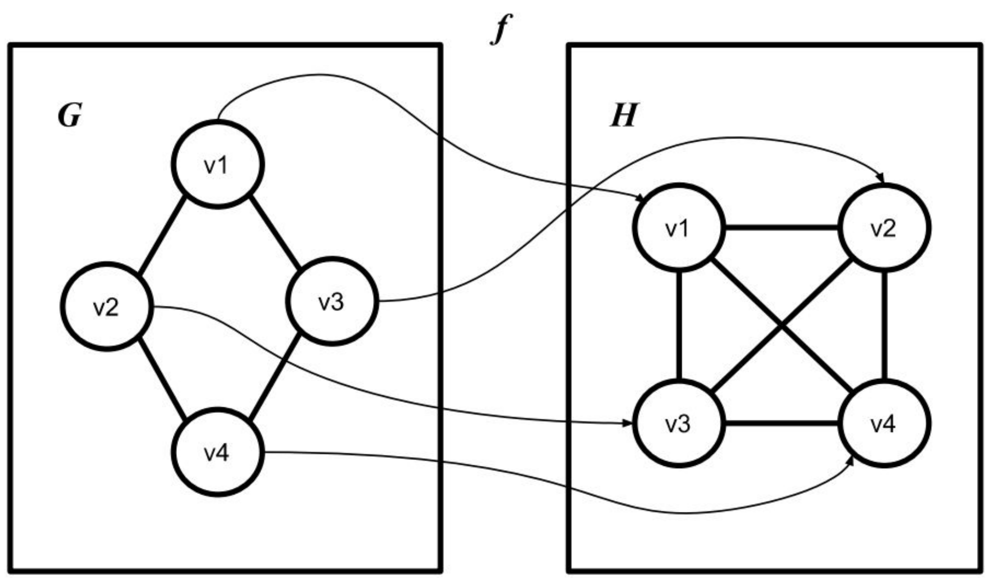
🤔 Is *f* a homomorphism?

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$



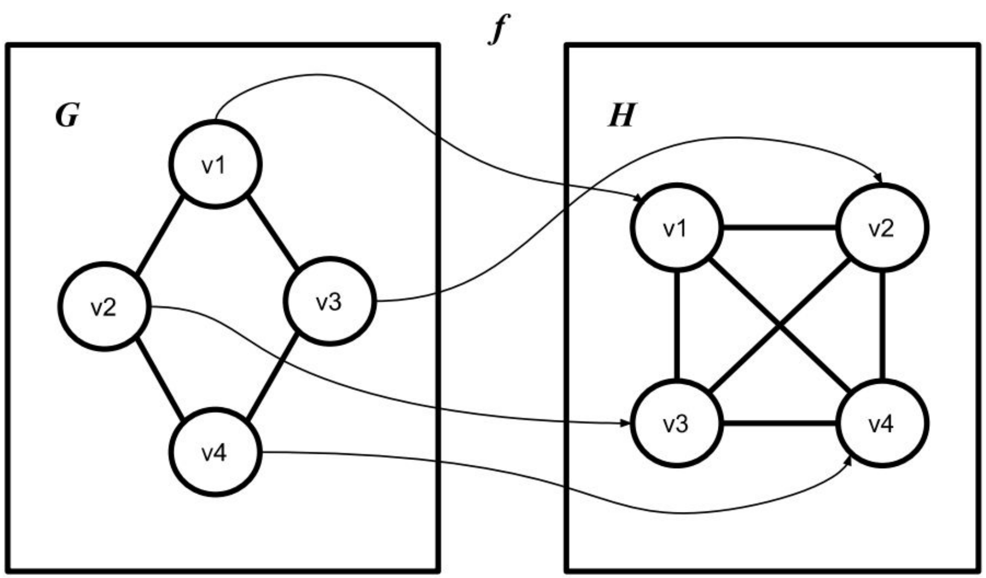
1. Trace the arrows from each pair of adjacent nodes

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$



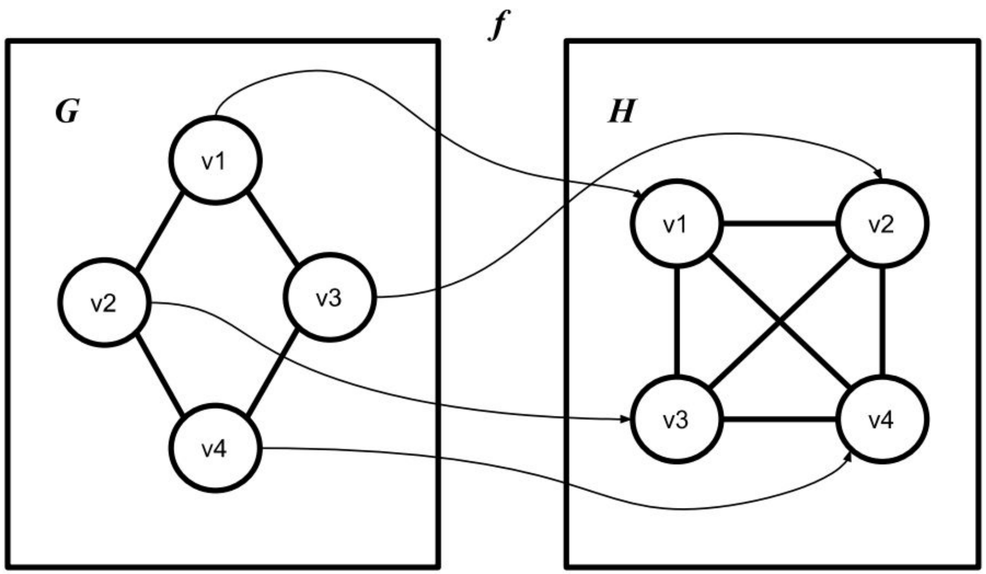
1. Trace the arrows from each pair of adjacent nodes
2. Check whether each pair remains adjacent in the “codomain graph”

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G. \forall v \in V_G. (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$



🤔 Is *f* a homomorphism? YES. 🤩

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G. \forall v \in V_G. (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

🤔 Do homomorphisms have special properties?

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

🤔 Do homomorphisms have special properties?

Are they…

Injective?

Surjective?

Bijective?

None of the above?!

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G. \forall v \in V_G. (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

🤔 Do homomorphisms have special properties?

Are they…

~~Injective?~~

~~Surjective?~~

~~Bijective?~~

None of the above!!

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

🤔 Do homomorphisms have special properties?

Are they…

~~Injective?~~

~~Surjective?~~

~~Bijective?~~

None of the above!!

(They are still special; they just don't have superpower.)

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

(5pts) Prove that for all graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$, and $G_3 = (V_3, E_3)$, if there exist homomorphisms $f : V_1 \to V_2$ and $g : V_2 \to V_3$, then there exists a homomorphism $h : V_1 \to V_3$.

"ASSUME" ---> *(which variables do we need here?)*

"W.T.S."------->

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G. \forall v \in V_G. (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

(5pts) Prove that for all graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$, and $G_3 = (V_3, E_3)$, if there exist homomorphisms $f : V_1 \to V_2$ and $g : V_2 \to V_3$, then there exists a homomorphism $h : V_1 \to V_3$.

"ASSUME" → Consider arbitrary graphs G1=(V1, E1), G2 =(V2, E2), G3=(V3,E3)...

"W.T.S."------->

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \to V_H$ where:

$$\forall u \in V_G.\ \forall v \in V_G.\ (\{u, v\} \in E_G \to \{f(u), f(v)\} \in E_H).$$

(5pts) Prove that for all graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$, and $G_3 = (V_3, E_3)$, if there exist homomorphisms $f : V_1 \to V_2$ and $g : V_2 \to V_3$, then there exists a homomorphism $h : V_1 \to V_3$.

"ASSUME" → Consider arbitrary graphs G1=(V1, E1), G2 =(V2, E2), G3=(V3,E3)
Such that there exist homomorphisms f: V1 → V2, g: V2 → V3.

"W.T.S."------->

Graph Theory & Functions

(Spring 2019, Midterm Practice 1) *Mighty Morphism Power Rangers*

First, we begin with a new definition—a special kind of function called a *homomorphism*, which operates on vertices of graphs. Specifically, given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, a homomorphism is a function $f : V_G \rightarrow V_H$ where:

$$\forall u \in V_G. \forall v \in V_G. (\{u, v\} \in E_G \rightarrow \{f(u), f(v)\} \in E_H).$$

(5pts) Prove that for all graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$, and $G_3 = (V_3, E_3)$, if there exist homomorphisms $f : V_1 \rightarrow V_2$ and $g : V_2 \rightarrow V_3$, then there exists a homomorphism $h : V_1 \rightarrow V_3$.

"ASSUME" → Consider arbitrary graphs G1=(V1, E1), G2 =(V2, E2), G3=(V3,E3)
Such that there exist homomorphisms f: V1 → V2, g: V2 → V3.

"W.T.S."-------> **There exists** a homomorphism h: V1 → V3.